

# IDIOSYNCRATIC RISK AND AUSTRALIAN EQUITY RETURNS

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## Abstract

In this paper we investigate the relationship between portfolio returns and idiosyncratic risk for Australian stocks. We report that the portfolio with highest idiosyncratic volatility generates an average annual return of over 45%. We observe additionally that the outcome is consistent with an exponential growth process for stock prices. Further, consistent with Malkiel and Xu, we observe that a stock's idiosyncratic volatility is inversely correlated with the size of the underlying firm. Thus, our model advances an interpretation of the Fama and French finding that portfolios of stocks of small firms offer superior risk-adjusted returns. Moreover, our findings challenge the portfolio theory of Markowitz (1959) and the asset-pricing model of Sharpe (1964).

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## 1. Introduction

In their article (Risk and Return Revisited, 1997), Malkiel and Xu, confirm the controversial finding of Fama and French (1992)<sup>1</sup> that beta does not appear as an explanatory variable when attempting to model the annual returns on US stocks from 1963 through 1990. In addition, they confirm the Fama and French finding of a clear tendency for the portfolios of smaller companies to produce rates of return that are greater than the returns from portfolios of larger companies. The revelation that the returns of firms with respect to beta remain essentially “flat” and that the size of a company appears to be a far better proxy for risk than beta naturally remain controversial.

Malkiel and Xu find additionally that portfolios of smaller companies have a higher idiosyncratic – or non-market correlated – volatility, and that portfolios of smaller companies post significantly higher average returns<sup>2</sup>. In this article, we confirm the findings of Malkiel and Xu in the context of Australian firms. In addition, we are able to advance an alternative explanation as to why the market might reward idiosyncratic risk. We advance the hypothesis that the portfolios of firms with higher idiosyncratic

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<sup>1</sup> Also see Fama and French (1993, 1995, 1996 and 1998)

<sup>2</sup> See Malkiel and Xu (2000) for the relationship between idiosyncratic risk and security returns. They observe that idiosyncratic volatility is more powerful than either beta or firm size effect in explaining the cross-section of stock returns. Also see Campbell et al (2001) for a detailed study of volatility of common stocks at the market, industry and firm levels.

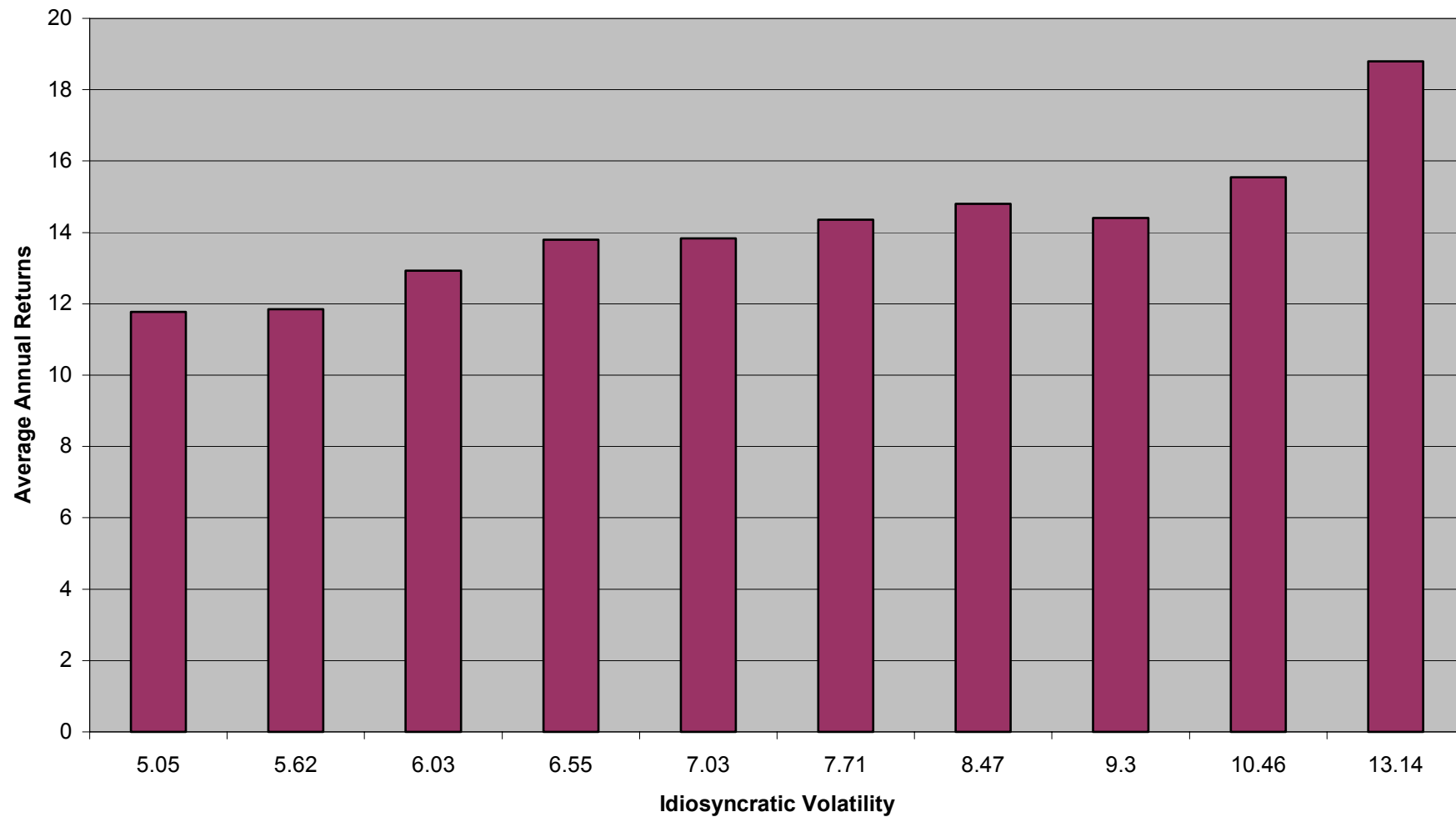
risk have higher returns, not because investors price them down in response to such risk, but because the higher risk *of itself* leads to higher returns.

As we show, such a view is consistent with an “organic” or exponential growth model of capital appreciation. In developing these ideas, the remainder of the paper is organized as follows. In section II, we present our findings for Australian firms with regard to our observed portfolio returns and the idiosyncratic risk of portfolio assets. Sections III and IV present our explanation of our findings. Thus we advance an organic or exponential growth model of capital appreciation. Here, we consider the implications of the model for portfolio diversification, and demonstrate that the predictions of the model are in accord with the empirical findings. Section V presents concluding comments.

## **2. The observed relationship between return and idiosyncratic risk**

Malkiel and Xu (1997) form ten portfolios of companies according to their idiosyncratic volatility. Over the thirty-one years from 1963 through 1994, the average annual returns display a monotonically increasing dependence on the portfolio’s idiosyncratic volatility. Thus the portfolio with lowest idiosyncratic volatility (5% per month) has an average annual return of just under 12%, while the portfolio with the highest idiosyncratic volatility (13% per month) has an average annual return of just under 19%. The results are summarized in Exhibit 1.0.

**Exhibit 1.0**  
**Relationship of Return and Idiosyncratic Volatility**  
**(Reporduced from Malkiel and Xu (1997))**



For our Australian sample of one hundred and twenty six firms from 1990 through 2000, four portfolios (P<sub>1</sub> through P<sub>4</sub>) were constructed due to limitations of sample size. Our analysis of idiosyncratic risk and return is restricted to firms with available returns data, on the Datastream<sup>3</sup> return files from 1990 through 2000. We adopt the approach of Malkiel and Xu (1997) in constructing portfolios of idiosyncratic volatility. Portfolio P<sub>1</sub> consists of firms with lowest idiosyncratic volatility and P<sub>4</sub> with highest idiosyncratic volatility. Our results display a remarkable similarity with those of Malkiel and Xu. Our portfolio with lowest idiosyncratic volatility (3.2% per month) has an average annual return of approximately 13.58% (monthly return: 1.132%). The second portfolio with idiosyncratic volatility of 7.3% per month has an average annual return of 16.90% (monthly return: 1.409%).

The third portfolio with idiosyncratic volatility of 11.4% per month has an average annual return of 19.83% (monthly return: 1.65%). These three portfolios cover the range of idiosyncratic risk observed by Malkiel and Xu. Our final portfolio with idiosyncratic volatility of 15.5% per month actually has an average annual return of approximately 45.5% (monthly return: 3.83%). The results are summarized in Table 1.0 and Exhibit 2.0.

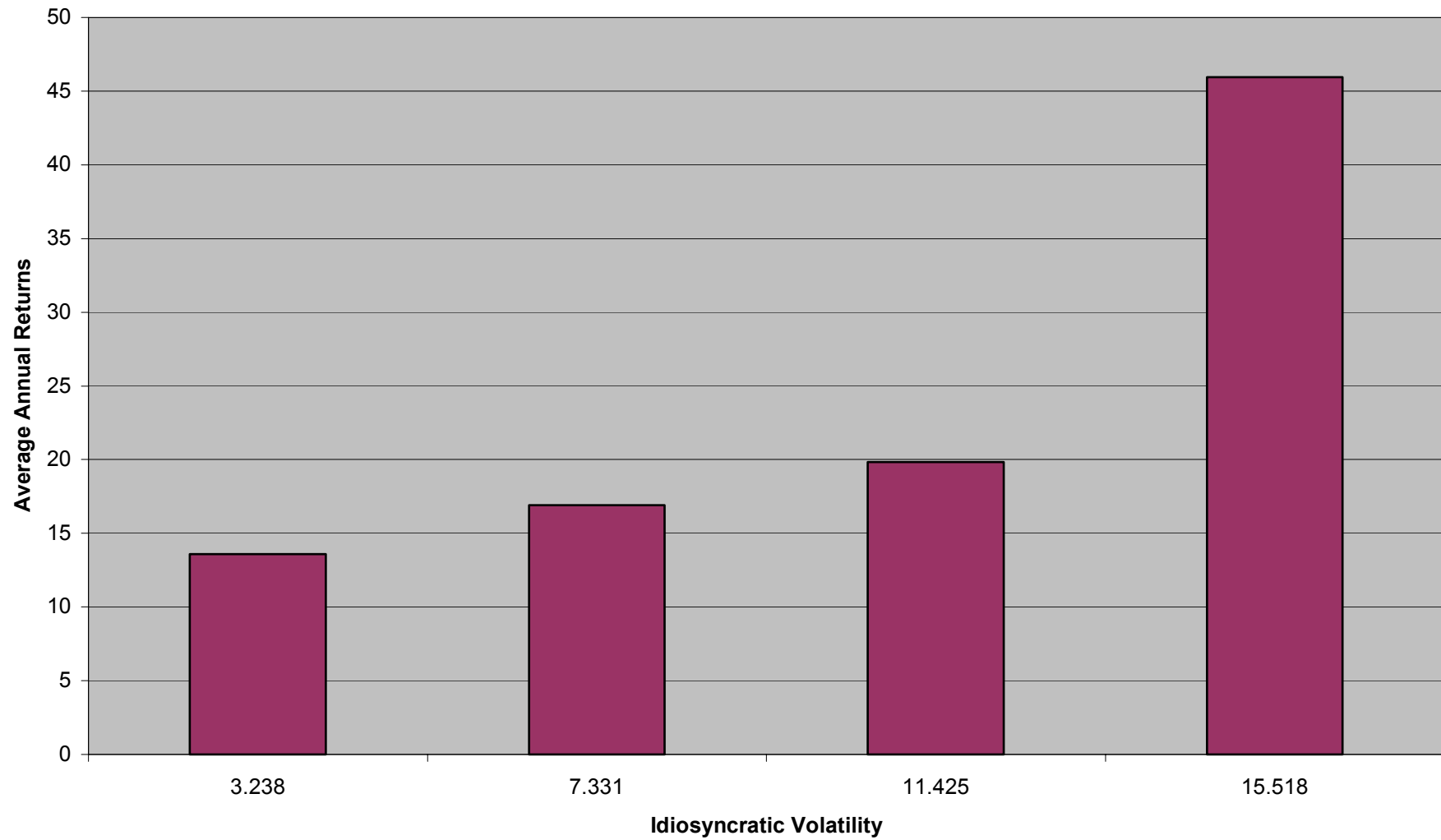
**Table 1.0**

<b>Portfolio</b>	<b>Idiosyncratic Volatility</b>	<b>Average Annual Returns</b>	<b>Sample Size</b>
<b>P<sub>1</sub></b>	<b>3.2%</b>	<b>13.584%</b>	<b>73</b>
<b>P<sub>2</sub></b>	<b>7.3%</b>	<b>16.908%</b>	<b>34</b>
<b>P<sub>3</sub></b>	<b>11.4%</b>	<b>19.836%</b>	<b>11</b>
<b>P<sub>4</sub></b>	<b>15.5%</b>	<b>45.948%</b>	<b>8</b>

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<sup>3</sup> Primark Corporation is a global information services company. We used Datastream, a Primark brand to obtain the data for this study.

**Exhibit 2.0**  
**Idiosyncratic Volatility and Return**



### 3. The organic or exponential growth process

Exponential or organic growth occurs commonly in nature and in the biological sciences. Such growth may be represented as the outcome of continuously applied growth rates that are selected independently across time from a normal distribution. For such growth, the outcome valuation at the end of a time period is the starting valuation multiplied by  $\exp(y)$  where  $y$  is normally distributed; or, stated alternatively, the starting valuation multiplied by  $\exp(\mu+x)$  where  $\mu$  represents the underlying mean or “drift” exponential growth rate for the period and  $x$  is normally distributed about zero with standard deviation (volatility),  $\sigma$ .

The assumption that stock market growth can be modelled by such a process is justified by the evidence of past stock price performance (for example, Fama, 1976, ch. 2; and more recently, Jones and Wilson, 1999); while Fama (1976) observes that an *a priori* expectation for such a process is reinforced by the mathematics of selection as captured by the Central Limit Theorem. Investors, however, assess stock prices not in terms of their potential exponential growth characteristics, but in terms of their “expected periodic return”, by which we mean the expected percentage increase in wealth generated over some *discrete* period (a month, a year). Such return is familiarly expressed:

$$\begin{aligned} \text{expected periodic return} = \\ [\text{expected wealth outcome of investing } X \text{ for a single period} - X] / X \end{aligned} \quad (1)$$

In order to generalize the relationship between the above *periodic* return – in effect, the “surface” return which market participants seek to measure - and the “sub-surface” *continuously-applied* parameters  $\mu$  and  $\sigma$  which generate the return, we commence by defining the “exponential growth rate for expected wealth” over a period,  $r$ , with the statement:

$$\$X \exp(r) = \text{expected wealth outcome of investing } \$X \text{ for a single period} \quad (2)$$

which provides:

$$r = \ln [\text{expected wealth outcome of investing } \$1 \text{ for a single period}] \quad (3)$$

where  $\ln$  represents the natural log function. Since the wealth outcome of investing \$1 for a single period is assumed to be distributed as  $\exp(y)$  where  $y$  is normally distributed with mean  $\mu$  and standard deviation  $\sigma$ , the above equation may be expressed:

$$r = \mu + \frac{1}{2} \sigma^2 \quad (4)$$

as utilized in the continuous time framework of the Black-Scholes model. Combining equations 1, 2 and 4, we arrive at the relationship between an investment’s expected periodic return and the parameters  $\mu$  and  $\sigma$  as:

$$\text{expected periodic return} = \exp(\mu + \frac{1}{2} \sigma^2) - 1 \quad (5)$$

Since  $\frac{1}{2} \sigma^2$  is necessarily positive, equation 5 confirms that the volatility of returns ( $\sigma$ ) in an exponential growth process necessarily acts to increase the expected periodic return.

To illustrate the above insights, consider an asset which, with equal likelihood, may grow at an exponential rate 15%, or, alternatively, decline at a rate 15%. Notwithstanding that the mean exponential growth rate ( $\mu$ ) is zero, the asset grows by an expected factor  $[\exp(0.15) + \exp(-0.15)] / 2 = 1.01127$ ; which is to say, with an “exponential growth rate for expected wealth”  $r$  (equation 3) equal to  $\ln(1.01127) = 0.01121$ , or 1.121%. Note that in this example, the volatility or standard deviation ( $\sigma$ ) of the allowed exponential growth rates is 15%. With outcome exponential growth rates *normally* distributed about zero with standard deviation 15%, equation 5 states that the expectation of return  $r$  is  $\frac{1}{2} (0.15)^2$ ; which is to say, 0.01125, or 1.125%.

With investment in a sufficiently large number of assets each with mean growth rate,  $\mu$ , and *idiosyncratic* volatility about such rate,  $\sigma$ , all growth rates may be assumed to occur in proportion to their probability of occurrence; in which case, the return generated by the idiosyncratic volatility,  $\frac{1}{2} \sigma^2$ , becomes a certain outcome. If the risk captured by the volatility  $\sigma$  is *systematic* across assets in the portfolio, the expectation of return generated by the volatility remains as  $\frac{1}{2} \sigma^2$ , but may, of course, exceed or fall short of such expectation.

The organic model of appreciation is consistent with the observation that the potential wealth outcome of an investment is, on the upside, theoretically unbounded, while on the downside it cannot be less than zero (since  $\exp(\infty) = \infty$ , while  $\exp(-\infty) = 0$ ). The model predicts that when volatility acts idiosyncratically to increase the number of unusually high returns, and simultaneously the number of unusually low returns in a portfolio, the overall outcome is an increase in the portfolio’s overall rate of return.

#### 4. Idiosyncratic Risk and the Empirical Evidence

Given that idiosyncratic risk is effectively diversified away in a portfolio that comprises a sufficient number of uncorrelated stocks, we might expect that, in equilibrium, the return generated by idiosyncratic volatility,  $\frac{1}{2} \sigma^2$ , should be eliminated by upward adjustments in the prices of stocks. It is likely, however, that such equilibrium did not dominate observed returns over the period of the above studies. Firstly, as Malkiel and Xu have observed, the idiosyncratic volatility of small firm stocks had been *increasing* over the period of their study in a manner that remained essentially *unrecognized* at the time. Secondly, in so far as prices over the period of the study might have been adjusted upward in response to the return generated by idiosyncratic risk, such upward adjustment – by virtue of being captured by the data – would have had the effect of actually increasing - rather than decreasing - the measured returns on these stocks.

The measured average annual return for the portfolios as a function of the portfolio’s idiosyncratic volatility displays an unambiguously increasing relationship (both those of Malkiel and Xu for the US stock markets, and our own for the Australian markets). It is possible to impose the curve  $r = \mu + \frac{1}{2} \sigma^2$  on these findings. Their findings appear to be consistent with the prediction that the increasing return of their portfolios should be as a quadratic function of idiosyncratic volatility. In our model, an idiosyncratic volatility of 5% per month contributes  $\frac{1}{2} \sigma^2 = 0.125\%$  to the monthly return (1.50% annualized); an idiosyncratic volatility of 13% per cent per month contributes  $\frac{1}{2} \sigma^2 = 0.845\%$  to the monthly return (10.14% annualized); while an idiosyncratic volatility of 15.5% per cent per month contributes  $\frac{1}{2} \sigma^2 = 1.20\%$  to the monthly return (14.42% annualized). In Figure 1.0, we have

displayed our model predictions against the empirical observations for the US and Australian markets.

## **5. Conclusion and implications**

The central prediction of the Capital Asset Pricing Model of Sharpe (1964) is that expected returns on securities are a positive linear function of their market betas. The CAPM also implies that there is a reward for bearing systematic risk which, is measured by the market risk premium. The CAPM also implies that idiosyncratic risk can be eliminated in a diversified portfolio and hence investors will not be rewarded for bearing idiosyncratic risks. However, Malkiel and Xu (1997 and 2000) contradict the CAPM by observing that idiosyncratic volatility is priced in the market and hence related to stock returns.

Our findings are: (a) portfolio with high idiosyncratic risks generates high returns, (b) small stocks high idiosyncratic risks and therefore generate high returns. Malkiel and Xu (1997) are of the view that the relationship between idiosyncratic risks and stocks returns contradict the tenets of the CAPM which are the foundation of our understanding of how rational markets are expected to function. In response, we have advanced a model of stock market growth in terms of exponential rather than discrete one-period growth as advanced by the traditional CAPM. Whereas the one-period model implies that growth rates of  $x\%$  and  $-x\%$  on equally-weighted investments effectively cancel, the organic growth model generates the growth factor  $[\exp(x) + \exp(-x)]/2$ , which is always greater than unity. Thus the organic model of capital appreciation predicts that given two well-diversified portfolios with the same mean exponential growth rate,  $\mu$ , the one whose assets have the higher idiosyncratic volatility will have the higher return.

The prediction is consistent with the empirical findings for both US and Australian stock markets as reported in this paper. As observed by Malkiel and Xu, the volatility of their stocks is correlated closely with the inverse of their firm size. Our findings therefore suggest a satisfying theoretical explanation for the findings of Fama and French to the effect that returns on stocks appear to be correlated closely with the inverse of their firm size. In addition, our findings challenge the portfolio theory of Markowitz (1952) and the CAPM of Sharpe (1964) which advances the notion that it is rational for a utility maximizing investor to hold a well-diversified portfolio of investments to eliminate idiosyncratic risks. In our view, a fascinating area of future research is to conduct additional empirical tests on the role of idiosyncratic risk in asset pricing. This is an issue we explore in our next paper.

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